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HEAT TRANSFER BETWEEN HEATED PLATES OF FINITE WIDTH IN A FREE MOLECULE FLOW ENVIRONMENT

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ABSTRACT

Results for the free molecule mass flow rate through a flat plate channel of finite length, assuming diffuse molecular reflections from the wall, are obtained and are in good agreement with previously obtained approximate results.

Results for the heat transfer between the channel walls and to the environment due to free molecule flow and thermal radiation when the channel walls and the environment at the inlet and exit are all at different temperatures are given.

The heat transferred by the free molecule flow is found to be significant when compared to thermal radiation for conditions that would be applicable in a thermionic converter.

INTRODUCTION

There is a general interest in the rarefied gas mode of heat transfer because of the low density environment that is being encountered in present day technology. When the mean free path of the molecules is large compared to the model dimensions the usual continuum transfer equations, which are really limiting solutions for very small mean free molecule paths, no longer apply. The problem must then be solved using the kinetic theory of gases.

The problem of free molecule flow heat transfer between infinite plates was treated by Knudson as discussed in (1). The problem of heat transfer in an adiabetic tube and nozzle with free molecule flow was treated in (2,3). The heat transfer to a nonconvex surface from a free molecule stream is treated in (4).

The model analyzed here consists of a flat plate channel of finite length with the plates at different temperatures and with a free molecule flow between them as shown in Figure 1. The right and left environments of the channel are at different temperatures and densities. First, the molecular stream incident on the surface of the channel wall will be calculated. From this the mass transfer flow rate through the channel formed by the plates is found. This will be compared to the results obtained by Clausing (5) using an approximate method of solution. Then the total energy leaving the surface is obtained. Using these results the net energy transferred between the surfaces and the environment is found. It is assumed that the wall temperature is isothermal along its length and that the accomodation coefficients are constant for both plates. Finally, the thermal radiation heat transfer is calculated for a similar model, and the heat transferred by radiation is compared to that for free molecule flow for a particular situation that might arise in thermionic energy converters.

NOTATION

A_1	= area of plate
A_L	= inlet cross-sectional area
c_v	= heat capacity at constant volume
E	= energy per unit mass of the molecular stream, $\left(c_v + \frac{R}{2}\right)T$

e = rate of energy per unit area leaving a surface

$e_{x0,t}$ = total energy emitted and reflected leaving surface 1 per unit area at location x_0

F_{x-L} = shape factor from left end to element at point x, Eq. (2)

f = mass flow ratio, $(m - m_R)/(m_L - m_R)$

K = kernal, Eq. (3)

l = length of plates divided by distance between them

M = weight of molecule

MW = molecular weight

m = mass flow rate per unit area

m_{L-R} = mass flow rate through the channel per unit cross-sectional area

N_V = number of molecules per unit volume with velocity in range V to $V + dV$ with a net velocity component in the direction of the positive normal

Q = total heat rate leaving surface

R = gas constant, $\frac{8.31\times10^7}{MW}\,\frac{\text{sq. cm.}}{\text{sq. sec. }^\circ K}$ or $\frac{1.987}{MW}\,\frac{\text{cal.}}{^\circ K\text{ g}}$

T = temperature

u = average nontranslational internal energy of the molecules

V = velocity of molecule

x = distance along lower plate divided by the distance between the plates

Greek Letters:

α = accomodation coefficient, emissivity

β = $1/(2RT)^{1/2}$, Eq. (A1)

ξ = distance along upper plate divided by distance between plates

ρ = density

ρ_s = density at standard conditions, 273° K and 1 atm

σ = Stefan Boltzmann constant, 1.36×10^{-12} cal./sq. cm. sec. $^\circ K^4$

φ = subsolutions as given by Eqs. (10) to (14)

$\overline{\varphi}$ = integrated mean, $\frac{1}{l}\int_0^l \varphi\, dx$

$\overline{\varphi F}$ = integral $\int_0^l \varphi F_{x-L}\, dx$

ψ = angle

Subscripts:

A-B = from A to B

L = left environment

R = right environment

t = total leaving surface including reflected and emitted streams

x = position along lower plate

ξ = position along upper plate

0 = location on surface

1 = lower plate

2 = upper plate

ANALYSIS

The model analyzed consists of two plates whose length divided by the distance between them is l. The plates are infinite in width and are at temperatures T_1 and T_2, respectively. The left and right environments are at temperatures T_L and T_R and densities ρ_L and ρ_R, respectively, as shown in Figure 1. It is assumed that there is a Maxwellian equilibrium in the left and right environment and that the density of the gas is sufficiently low so that the mean free path of the molecules is large and therefore the effect of intermolecular collisions in the channel can be neglected. First, the total number of molecules incident on any part of the wall must be calculated.

Mass Flow Rate Leaving The Wall

The total mass flow per unit wall area m_{x0} reaching a surface element at point x_0 due to the direct influx of molecules from the open left and right ends and by rebounding molecules from the opposite wall is

$$m_{x0} = m_L F_{x0-L} + m_R F_{x0-R} + \int_{\xi=0}^{l} m_\xi K(x_0, \xi) d\xi \qquad (1)$$

The term m_L is the mass flow rate of molecules entering through the open left end per unit area and is derived in the appendix as equal to $m_L = \rho_L (RT_L/2\pi)^{1/2}$. The term F_{x-L} is the usual shape factor used in radiation that gives the fraction of molecules entering the channel times the entrance area that are incident on a unit area of the wall at location x_0. The term m_ξ is the mass flow rate of molecules per unit area leaving the upper wall at location ξ and is equal to the mass flow rate incident at that point. It is assumed that the molecules leave the surface in a diffuse manner. The term $K(x, \xi)$ is the shape factor that gives the fraction of the emitted molecules from the infinite strip of differential width at ξ times the area of this strip that reach the elemental area at x per unit area. These are given by

$$F_{x-L} = \frac{1}{2}\left[1 - \frac{x}{(x^2 + 1)^{1/2}}\right] ; \; F_{x-R} = \frac{1}{2}\left\{1 - \frac{l - x}{\left[(l - x)^2 + 1\right]^{1/2}}\right\} \tag{2}$$

$$K(x,\xi) = \frac{1}{2\left[(\xi - x)^2 + 1\right]^{3/2}} \tag{3}$$

and are derived from the interchange factor given on page 21 of Jakob (6). The similarity of equation (1) to the equation for radiant heat transfer is discussed in (2).

If all the m's in Equation (1) is equated to m_R, the resulting relationship will be correct as can be seen by the integration of the intergral term. This corresponds to the case of an isothermal enclosure where the rate of molecules per unit area leaving any surface of the enclosure is the same. Adding this to Equation (1) gives after some rearrangement

$$\frac{m_{x0} - m_R}{m_L - m_R} \equiv f_{x0} = F_{x0-L} + \int_0^l f_\xi K(x_0,\xi)\,d\xi \tag{4}$$

From the symmetry of the problem, at equal distances along the plates $m_{x0} = m_{\xi 0}$ or $f_x = (f_\xi)_{\xi=x}$. The function f_x is antisymmetric, $f(x) = 1 - f(l - x)$, as can be proved by following a similar procedure as discussed in (2). The solution to Equation (4) was obtained by numerical integration and iteration. The results are shown in Figure 2 for various values of l.

Mass Flow Rate Through The Flat Plate Channel

The net mass flow rate through the channel per unit entrance area can be found from

$$m_{L-R} = m_L - 2\int_0^l m_x F_{x-L}\,dx - \left(m_R - 2\int_0^l m_R F_{x-R}\,dx\right) \tag{5}$$

The first term on the right is the mass flow rate per unit entrance area entering the channel, the second term is the mass flow rate leaving through the entrance that comes from the walls. The shape factor F_{x-L} gives, by use of the reciprocity rule for shape factors, $F_{L-x}\,dA_L = F_{x-L}\,dA_x$, the fraction of the mass rate leaving the walls per unit wall area that is transported out through the entrance of the channel times the entrance area of the channel. The mass rate leaving from the upper wall through the inlet is equal to the amount leaving from the lower wall by symmetry. The last two terms give the mass rate leaving through the inlet that enters the right end and has no collision with the wall. Since $\int_0^l F_{x-L}\,dx$ is equal to $\int_0^l F_{x-R}\,dx$ Eq. (5) can be rewritten as

$$\frac{m_{L-R}}{m_L - m_R} = 1 - 2\int_0^l f_x F_{x-L}\,dx \tag{6}$$

The mass flow rates through the channels are shown in Figure 3. It can be seen that the net through flow decreases as the length becomes larger. These results are compared to the solution of Clausing as given in (5) which was obtained by assuming a linear form of f_x with an added correction factor and are in good agreement with the more exact present solution.

Total Energy Leaving The Surface Element

The total energy per unit area $e_{x0,t}$ leaving surface 1 at point x_0 is derived in the same manner as Eq. (27) in (2). The accommodation coefficient is defined as

$$\alpha = \frac{e_{x0,t} - (e_{x0,t})_{\text{incident}}}{m_{x0}E_x - (e_{x0,t})_{\text{incident}}} \tag{7a}$$

where $(e_{x0,t})_{\text{incident}}$ is the total energy incident on the surface per unit area and $m_{x0}E_x$ is the rate at which the energy would be carried away from the surface if all the incident molecules achieved thermal equilibrium with the wall. This can be rewritten as

$$e_{x0,t} = \alpha m_x E_1 + (1 - \alpha)\left(m_L E_L F_{x0-L} + m_R E_R F_{x0-R} + \int_0^l e_{\xi,t} K(x_0, \xi) d\xi\right) \tag{7b}$$

Where E_L is the energy per unit mass of the stream entering from the left end and is derived in the appendix. This assumes accommodation coefficients α are equal for both isothermal walls and all the molecules are assumed to leave the walls diffusely.

If it is assumed that $e_{x0,t}$, mE_1, $m_R E_R$, and $e_{\xi,t}$ are equal to e_R in Equation (7a), this relationship is still true as can be proved by integration of the integral term and adding this to Equation (7b) to give

$$e_{x0,t} - e_R = \alpha(m_x E_1 - e_R) + (1 - \alpha)\left[(e_L - e_R)F_{x0-L} + \int_0^l (e_{\xi,t} - e_R)K(x_0, \xi)d\xi\right] \tag{8}$$

similarly for wall 2,

$$e_{\xi 0,t} - e_R = \alpha(m_{\xi 0}E_2 - e_R) + (1 - \alpha)\left[(e_L - e_R)F_{\xi 0-L} + \int_0^l (e_{x,t} - e_R)K(x, \xi_0)dx\right] \tag{9}$$

Because of the linearity of the problem, the principle of superposition can be used to reduce the problem into simpler parts that can be added together for various boundary conditions as follows:

$$e_{x,t} - e_R = \varphi_{1-L}(e_L - e_R) + \varphi_{1-1A}E_1(m_L - m_R) + \varphi_{1-1B}(m_R E_1 - e_R) + \varphi_{1-2A}E_2(m_L - m_R) + \varphi_{1-2B}(m_R E_2 - e_R) \qquad (10)$$

$$e_{\xi,t} - e_R = \varphi_{1-L}(e_L - e_R) + \varphi_{1-2A}E_1(m_L - m_R) + \varphi_{1-2B}(m_R E_1 - e_R) + \varphi_{1-1A}E_2(m_L - m_R) + \varphi_{1-1B}(m_R E_2 - e_R) \qquad (11)$$

where

$$\varphi_{1-L} = (1 - \alpha)\left[F_{x-L} + \int_0^l \varphi_{1-L}K(x,\xi)d\xi\right] \qquad (12)$$

$$\left.\begin{aligned} \varphi_{1-1A} &= \alpha f_x + (1 - \alpha)\int_0^l \varphi_{1-2A}K(x,\xi)d\xi & (13a)\\ \varphi_{1-2A} &= (1 - \alpha)\int_0^l \varphi_{1-1A}K(x,\xi)dx & (13b)\end{aligned}\right\}$$

$$\left.\begin{aligned} \varphi_{1-1B} &= \alpha + (1 - \alpha)\int_0^l \varphi_{1-2B}K(x,\xi)d\xi & (14a)\\ \varphi_{1-2B} &= (1 - \alpha)\int_0^l \varphi_{1-1B}K(x,\xi)dx & (14b)\end{aligned}\right\}$$

These subsolutions have some physical significance. If T_2 and T_L are equal to T_R while the lower surface is at T_1, and if $m_L = m_R$, that is, the densities as well as the temperatures of the left and right environment are equal, then for this case the total energy leaving the lower surface is $e_{x,t} = e_R + \varphi_{1-1B}(m_R E_1 - e_R)$ and the energy leaving the upper surface is $e_{\xi,t} = e_R + \varphi_{1-2B}(m_R E_1 - e_R)$. The curves for φ_{1-1B}

and φ_{1-2B} are given in Figure 4 for various values of l. It can readily be seen that the limiting cases for $l = 0$ are $(\varphi_{1-1B})_{l\to 0} = \alpha$ and $(\varphi_{1-2B})_{l\to 0} = 0$ and the limiting cases for $l = \infty$ are $(\varphi_{1-1B})_{l\to\infty} = \frac{1}{2 - \alpha}$ and $(\varphi_{1-2B})_{l\to\infty} = \frac{1 - \alpha}{2 - \alpha}$. The results are symmetrical around $x/l = 0.5$. In the limit as $\alpha \to 0$ the solution reduces to $\varphi_{1-1B} = \varphi_{1-2B} = 0$, while in the limiting case of $\alpha = 1$ the solution reduces to $\varphi_{1-1B} = 1$ and $\varphi_{1-2B} = 0$.

Similarly, if $T_1 = T_2 = T_R$, $m_L = m_R$, and $T_L \neq T_R$, the energy leaving either surface is given by $e_{x,t} = e_R + \varphi_{1-L}\,(e_L - e_R)$. These results are given in Figure 5. The limiting solution when $l = 0$ is $(\varphi_{1-L})_{l\to 0} = \frac{(1 - \alpha)}{2}$ and for $l \to \infty$ is $(\varphi_{1-L})_{l\to\infty} = 0$. The limiting solution for $\alpha = 1$ is $\varphi_{1-L} = 0$; the limiting solution for $\alpha = 0$ is the same as Equation (4) and is given in Figure 2.

Finally, if $T_1 = T_2 = T_R$ and if $e_L = e_R$ but $m_L \neq m_R$, then the total energy leaving either surface is given by $e_{x,t} = e_R + (m_L - m_R)E_R(\varphi_{1-1A} + \varphi_{1-2A})$. The results for φ_{1-1A} and φ_{1-2A} are shown in Figure 6. For $l = 0$ the solution reduce to $\varphi_{1-1A} = \alpha/2$ and $\varphi_{1-2A} = 0$.

Net Energy Leaving Surface

The net energy leaving surface 1 is the difference between the emitted and absorbed energy:

$$\frac{Q_1}{A_1} = \frac{\alpha E_1}{l} \int_0^l m_x \, dx - \frac{\alpha}{l} \int_0^l (e_{x,t})_{\text{incident}} \, dx \tag{15}$$

Since

$$e_{x,t} = (1 - \alpha)(e_{x,t})_{incident} + \alpha m_x E_1 \tag{16}$$

Equation (15) becomes

$$\frac{Q_1}{A_1} = \frac{\alpha}{(1 - \alpha) l} \int_0^l (E_1 m_x - e_{x,t} + e_R - e_R) dx \tag{17}$$

Since $\frac{1}{l} \int_0^l f_x \, dx = \frac{1}{2}$ because of the antisymmetry, Equation (17) becomes

$$\frac{Q_1}{A_1} \frac{(1 - \alpha)}{\alpha} = E_1 \left(\frac{m_L + m_R}{2}\right) - e_R - \left(\overline{e_{x,t} - e_R}\right) \tag{18}$$

where the bar denotes the integrated average value. Similarly, the net heat leaving wall 2 would be given by

$$\frac{Q_2}{A_1} = \frac{\alpha}{(1 - \alpha) l} \int_0^l (E_2 m - e_{\xi,t}) d\xi \tag{19}$$

or

$$\frac{Q_2}{A_1} \frac{(1 - \alpha)}{\alpha} = E_2 \left(\frac{m_L + m_R}{2}\right) - e_R - \left(\overline{e_{\xi,t} - e_R}\right) \tag{20}$$

The integrated values $\overline{\varphi} = \frac{1}{l} \int_0^l \varphi \, dx$ needed to evaluate $\left(\overline{e_{x,t} - e_R}\right)$ from Equations (10) and (11) are given in Figure 7.

Net Energy from the Environment

The net energy entering the channel through the left end is equal to

$$\frac{Q_L}{A_L} = m_L E_L - \int_0^l e_{x,t} F_{x-L} \, dx - \int_0^l e_{\xi,t} F_{\xi-L} \, d\xi$$

$$- m_R E_R \left(1 - 2 \int_0^l F_{x-R} \, dx\right) \tag{21}$$

which can be written as

$$\frac{Q_L}{A_L} = m_L E_L - m_R E_R - \int_0^l (e_{x,t} - e_R) F_{x-L}\, dx - \int_0^l (e_{\xi,t} - e_R) F_{\xi-L}\, d\xi \qquad (22)$$

which is equal to

$$\frac{Q_L}{A_L} = e_L - e_R - \left[\overline{(e_{x,t} - e_R) F_{x-L}}\right] - \left[\overline{(e_{\xi,t} - e_R) F_{\xi-L}}\right] \qquad (23)$$

Shown in Figure 8 are the integrated results for $\overline{\varphi_{1L}F}$, $\overline{\varphi_{1-1A}F}$, $\overline{\varphi_{1-2A}F}$, $\overline{\varphi_{1-2B}F}$, and $\overline{\varphi_{1-1B}F}$ where $\overline{\varphi F} = \int_0^l \varphi F\, dx$. Using these terms with Equations (10) and (11) gives Q_L/A_L.

by conservation of energy the net energy from the right side can be obtained from

$$\frac{Q_L}{lA_L} + \frac{Q_R}{lA_L} + \frac{Q_1}{A_1} + \frac{Q_2}{A_1} = 0 \qquad (24)$$

RADIATION HEAT TRANSFER

In most cases where the free molecule flow is important the thermal radiation will also be important. It is of interest, therefore, to give the radiation results. These can be obtained from equations similar to those for the free molecule flow. The radiation problem for the model treated here is similar to one treated in (7), but it includes the effects of the right and left environments that in (7) were neglected. From an energy balance the total energy leaving point x_0 by radiation for the present model is

$$e_{x0,t} = \alpha e_1 + (1 - \alpha)\left(e_L F_{x0-L} + e_R F_{x0-R} + \int_0^l e_{\xi,t} K(x_0, \xi)\, d\xi\right) \qquad (25)$$

In this case $e_{x0,t}$ is the total energy leaving point x_0 by radiation and e_L and e_R for this case are equal to σT_L^4 and σT_R^4. The emissivity of the surfaces is given by α. Equation (25) can also be written as

$$e_{x0,t} - e_R = \alpha(e_1 - e_R) + (1 - \alpha)\left[(e_L - e_R)F_{x0-L} + \int_0^l (e_{\xi,t} - e_R)K(x_0,\xi)d\xi\right] \tag{26}$$

A similar equation would apply for wall 2:

$$e_{\xi 0,t} - e_R = \alpha(e_2 - e_R) + (1 - \alpha)\left[(e_L - e_R)F_{\xi 0-L} + \int_0^l (e_{x,t} - e_R)K(x,\xi_0)dx\right] \tag{27}$$

Since this equation is linear, by superposition, it can be reduced to simpler parts as follows:

$$e_{x,t} - e_R = \varphi_{1-1B}(e_1 - e_R) + \varphi_{1-2B}(e_2 - e_R) + \varphi_{1L}(e_L - e_R) \tag{28a}$$

and

$$e_{\xi,t} - e_R = \varphi_{1-2B}(e_1 - e_R) + \varphi_{1-1B}(e_2 - e_R) + \varphi_{1L}(e_L - e_R) \tag{28b}$$

where the φ's are the same relationships given in Eqs. (12) and (14).

The net heat radiated from wall 1 can be calculated similarly to the free molecule case to be

$$\frac{Q_1}{A_1} = \frac{\alpha}{(1 - \alpha)l} \int_0^l (e_1 - e_{xt})dx \tag{29}$$

which gives

$$\frac{Q_1}{A_1}\frac{(1 - \alpha)}{\alpha} = e_1 - e_R - \left(\overline{e_{x,t} - e_R}\right) \tag{30}$$

The term $e_{x,t} - e_R$ is obtained by finding the integrated average value of Eq. (28) by the use of Figure 7.

Similarly for wall 2,

$$\frac{Q_2}{A_1}\frac{(1 - \alpha)}{\alpha} = e_L - e_R - \overline{\left(e_{\xi,t} - e_R\right)} \tag{31}$$

The energy entering from the left end is also obtained as before:

$$\frac{Q_L}{A_L} = e_L - e_R - \int_0^l (e_{x,t} - e_R)F_{x-L}\, dx - \int_0^l (e_{\xi,t} - e_R)F_{\xi-L}\, d\xi \tag{32}$$

which can be written as

$$\frac{Q_L}{A_L} = e_L - e_R - \overline{(e_{x,t} - e_R)F_{x-L}} - \overline{(e_{\xi,t} - e_R)F_{\xi-L}} \tag{33}$$

By the use of Equation (24) Q_R can then also be found.

Example Case

For purposes of illustration and to indicate the magnitude of the free molecule heat transfer an example is carried out. Consider the case where the left and right environments and one wall are at equal temperatures and the densities of the left and right environments are equal while the other wall is at temperature T_1. This type of situation may arise in a thermionic energy convertor. For this case the free molecule heat transfer from Equations (10) and (18) is seen to be

$$\left[\frac{Q_1}{A_1}\right]_{conv} = \left[\frac{\alpha m_R(E_1 - E_R)}{(1 - \alpha)}\right]_{conv} (1 - \overline{\Phi}_{1-1B}) \tag{34}$$

Similarly for radiation, from Eqs. (28) and (30)

$$\left[\frac{Q_1}{A_1}\right]_{rad} = \left[\frac{\alpha(e_1 - e_R)}{1 - \alpha}\right]_{rad} (1 - \overline{\varphi}_{1-1B}) \tag{35}$$

Taking a ratio gives

$$\frac{Q_{1,conv}}{Q_{1,rad}} = \left[\frac{1 - \alpha}{\alpha(e_1 - e_R)}\right]_{rad} \left[\frac{\alpha m_R(E_1 - E_R)}{1 - \alpha}\right]_{conv} \tag{36}$$

The gas is taken as argon at a density of 10^{-4} ρ_s where ρ_s is the density at standard conditions (273° K and 1 atm), and the plates are tungsten with plate 1 at 2000° K and plate 2 at 500° K. The wall emissivity is taken equal to 0.3 and therefore

$$\left[\frac{1 - \alpha}{\alpha\sigma(T_1^4 - T_R^4)}\right]_{rad} = 0.107 \ \frac{\text{sq. cm. sec.}}{\text{cal.}}$$

The accommodation coefficient for the argon-tungsten combination is given in (8) as 0.85. Also

$$m_R = \rho_R \left(\frac{RT_R}{2\pi}\right)^{1/2} = 2.296\times10^{-3} \ \frac{\text{gr.}}{\text{sq. cm. sec.}}$$

Since for argon $c_v = \frac{3}{2} R$,

$$E_1 - E_R = 2R(T_1 - T_R) = 149 \text{ cal./g.}$$

Combining these results gives

$$\frac{Q_{1,conv}}{Q_{1,rad}} = 0.21$$

which indicates that the free molecule flow heat transfer is not negligible for conditions that might occur in a thermionic device. This ratio, however, will depend strongly on the conditions chosen. For [redacted] higher wall temperatures than chosen here the radiation heat transfer will increase because the radiation depends on the temperature to the fourth power. For higher densities the free molecular heat transfer will increase since it is directly proportional to the density. At very high densities however, the present solutions are no longer applicable because the mean free path will be small compared to the channel width, and the effect of intermolecular collisions will become important.

The dimension of the channel for free molecule flow to occur can be found if we know the mean free path of the gas molecule. For hard sphere molecules the mean free path L_m is given by

$$L_m = \frac{1}{\sqrt{2}\ \pi m \sigma_m^2}$$

where σ_m is the molecular cross section. Table 1.6 in (8) gives for argon

$$L_m = 6.2\times10^{-6}\ \frac{\rho_s}{\rho}\ \text{cm.}$$

For ρ_s/ρ of 10^4, L_m is 0.062 cm. which is large compared to distance between the plates used in thermionic energy convectors.

RESULTS AND CONCLUSIONS

The present results can be used to find the mass flow through a channel in a free molecule environment, and they are in good agreement with the approximate solution of Clausing. The heat transfer between the surfaces and the environment by free molecule flow and by thermal radiation can be found for arbitrary combinations of temperatures by superposition of simple solutions. A comparison of the radiation heat transfer to the free molecule heat transfer in a sample case that would arise in a thermionic converter shows that the free molecule heat transfer can be significant when compared to the radiative heat transfer.

APPENDIX - MASS FLOW THROUGH AN AREA ELEMENT

The mass flow rate through an elemental area dA with a Maxwellian gas below it can be calculated as follows. Below dA there are N_V molecules per unit volume with velocities in the range of V to V + dV, and all possess a velocity component in the direction of the positive normal on dA. Assuming a Maxwellian distribution,

$$N_V = \frac{2\rho}{M} \frac{\beta^3}{\pi^{1/2}} V^2 e^{-\beta^2 V^2} \, dV \tag{A1}$$

where ρ is the local gas density, β equals $\frac{1}{(2RT)^{1/2}}$, and M is the mass of a gas molecule. Since the molecules are traveling equally in all directions in the hemisphere, the number moving in the direction ψ in the solid angle $d\omega$ is $N_V \frac{d\omega}{2\pi}$. Of these, the number of molecules in the volume of slant height $V \Delta t$ and area dA will cross dA in time Δt. These can be written as $N_V \frac{d\omega}{2\pi} V \cos \psi \, dA \, \Delta t$. Thus, the molecules through dA are distributed according to Lamberts cosine law and $\frac{\cos \psi \, d\omega}{\pi} dA$ can be written as $F_{dA-d\omega} \, dA$, which is the same shape factor used in thermal radiation calculations. Integrated over all velocities to give the mass flow leaving dA in the solid angle $d\omega$ gives $mF_{dA-d\omega} \, dA$ where m is

$$m = \rho \left(\frac{RT}{2\pi}\right)^{1/2} \tag{A2}$$

The m can be considered as the total mass flow rate crossing the surface per unit area.

The energy of the molecular stream through dA can be calculated as follows. The number of molecules crossing dA per unit time per unit area is $N_V V/2$ as obtained by integrating the number crossing a solid angle over the upper hemisphere. The energy of each molecule in the reservoir is $1/2\ MV^2 + MU$ where U is the average nontranslational internal energy of the molecules in the reservoir enclosure. Then the energy crossing dA per unit area per unit time can be integrated over all V to give

$$\int_0^\infty \frac{\rho\beta^3}{\pi^{1/2}} V^3 e^{-\beta^2 V^2} \left(\frac{V^2}{2} + U\right) dV = \frac{1}{2\pi^{1/2}} \rho \left(\frac{1}{\beta^2} + U\right) \tag{A3}$$

Since $U = \left(C_V - \frac{3}{2} R\right) T$, ref. (1), Equation (A3) becomes

$$\rho \left(\frac{RT}{2\pi}\right)^{1/2} \left(C_V + \frac{R}{2}\right) T = mE \tag{A4}$$

where E is the energy per unit mass of the Maxwellian stream crossing dA.

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E-2215

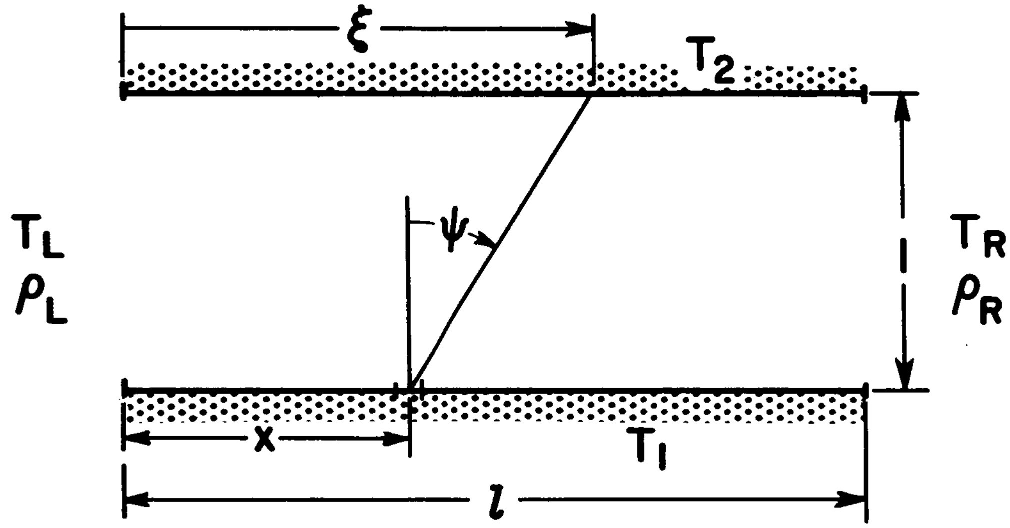


Fig. 1. - Analytical model.

E-2215

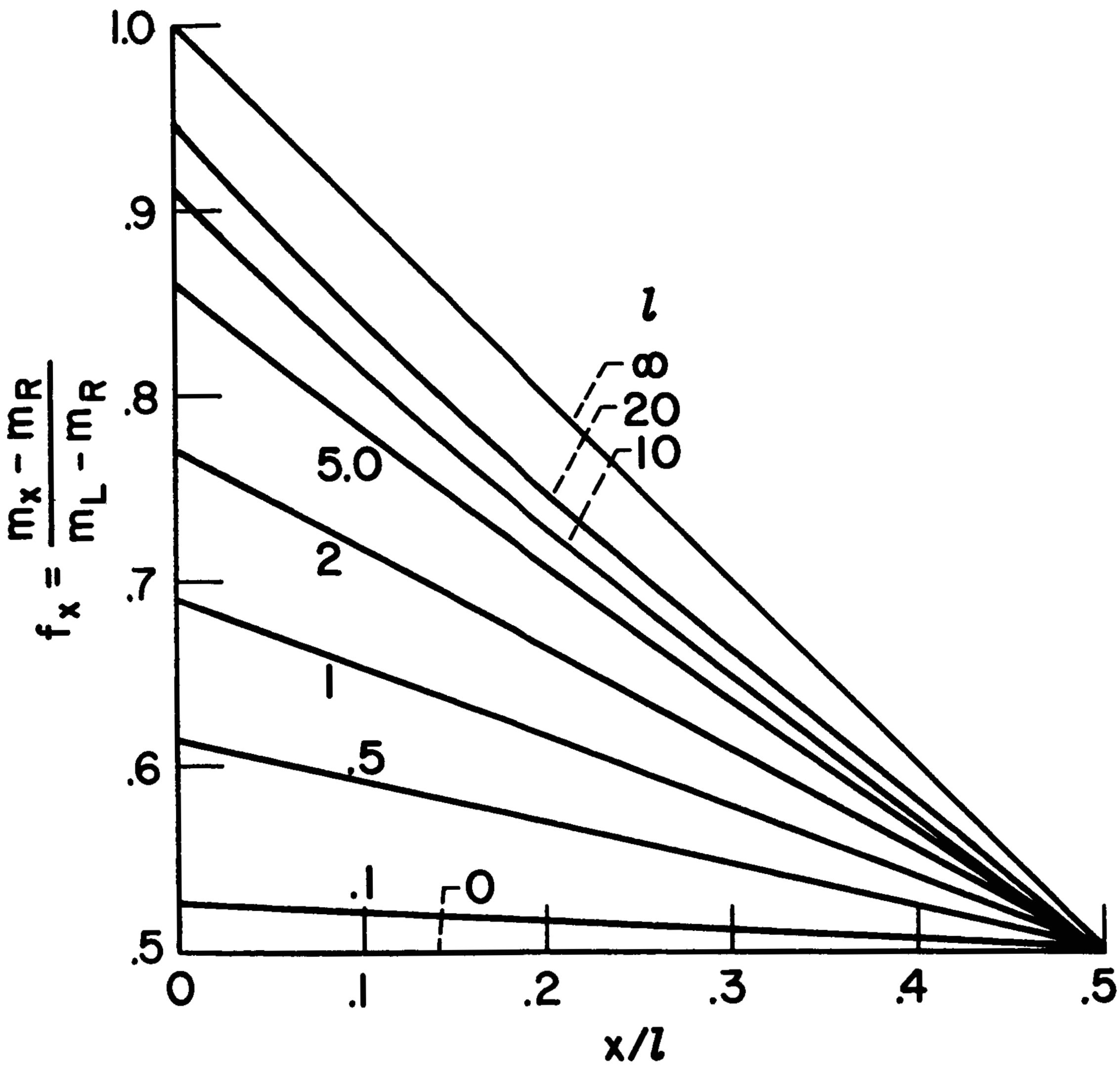


Fig. 2. - Mass flow ratio from the surface.

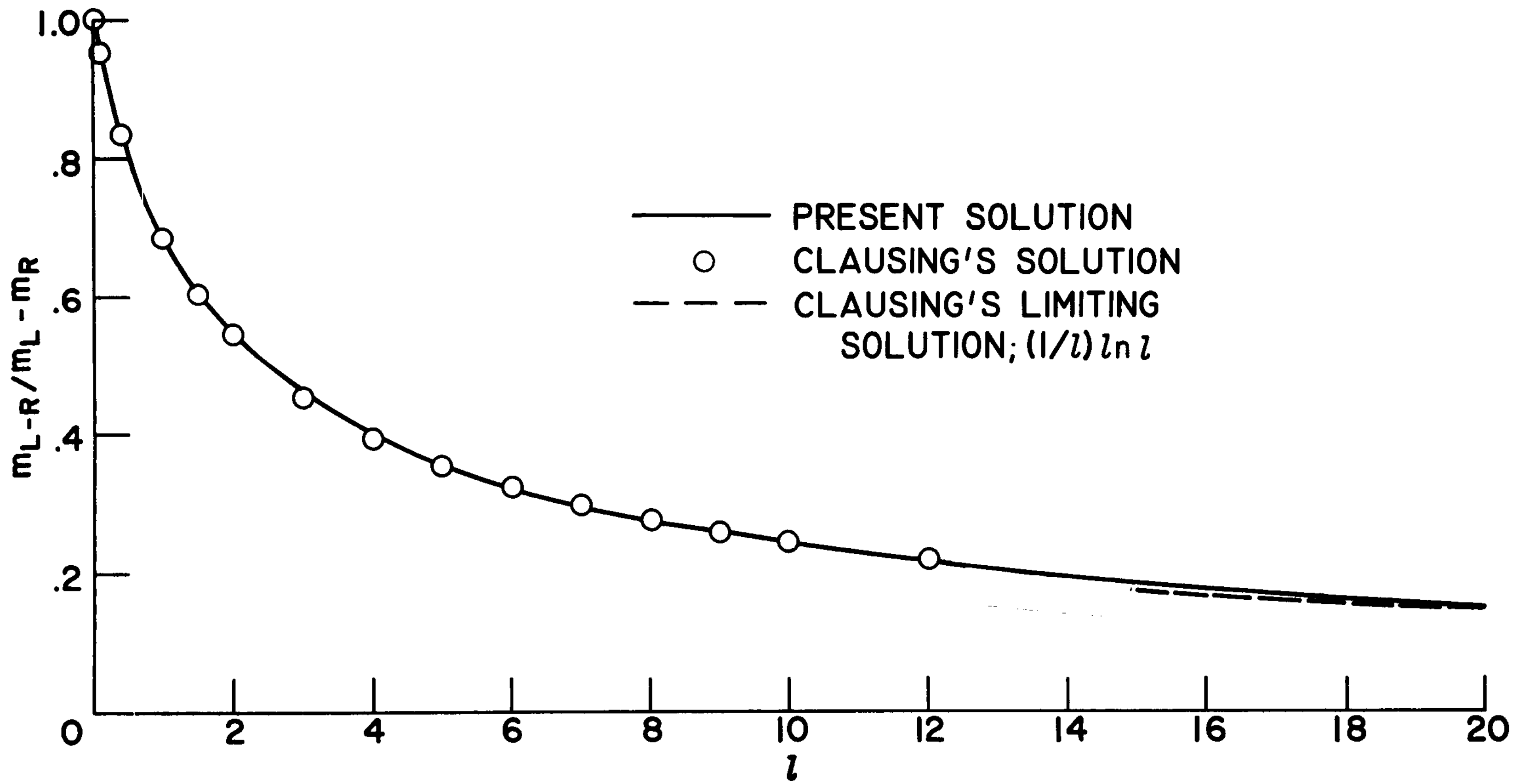
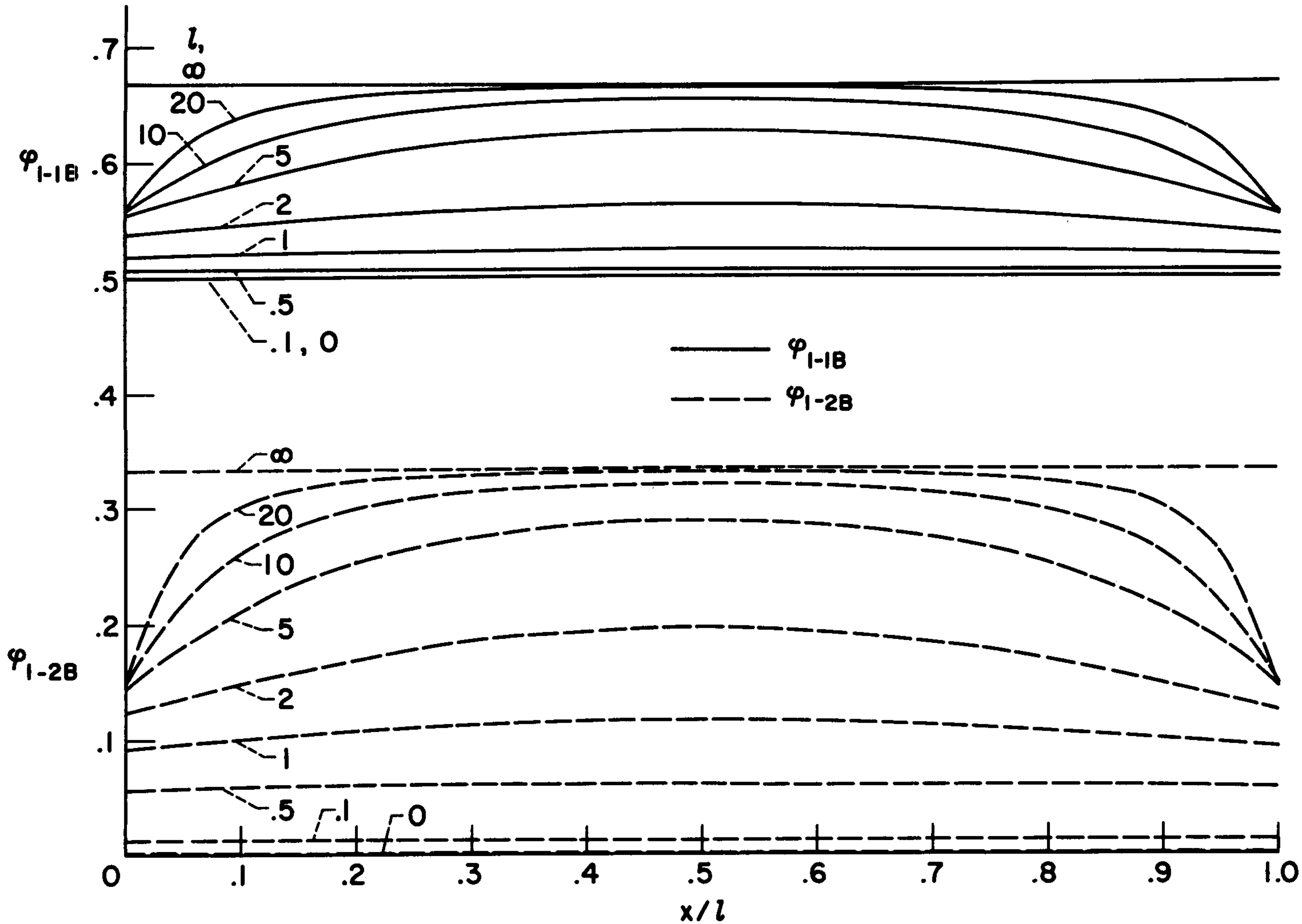
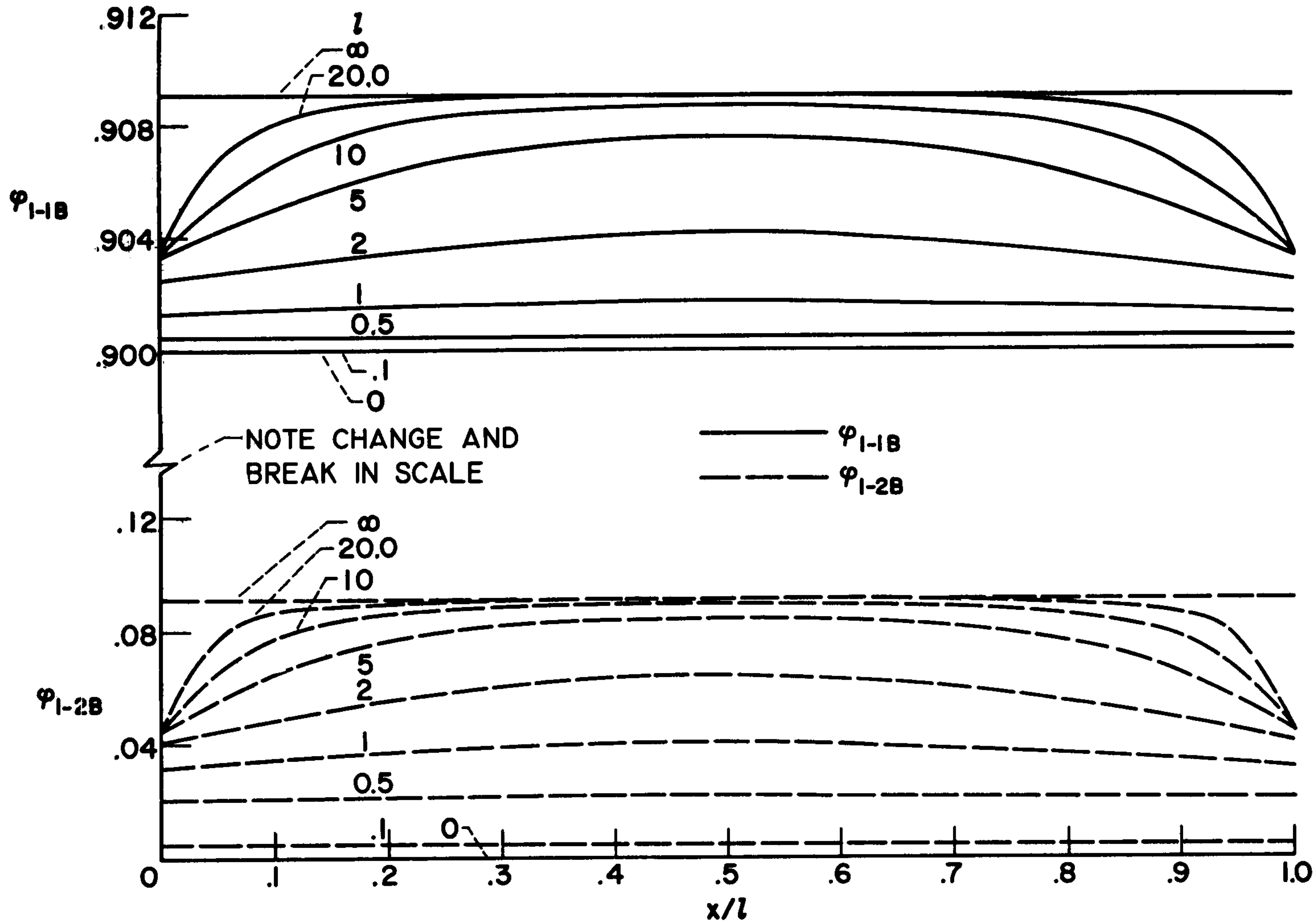


Fig. 3. - Mass flow ratio through the channel.



(a) Accommodation coefficient, $\alpha = 0.5$.

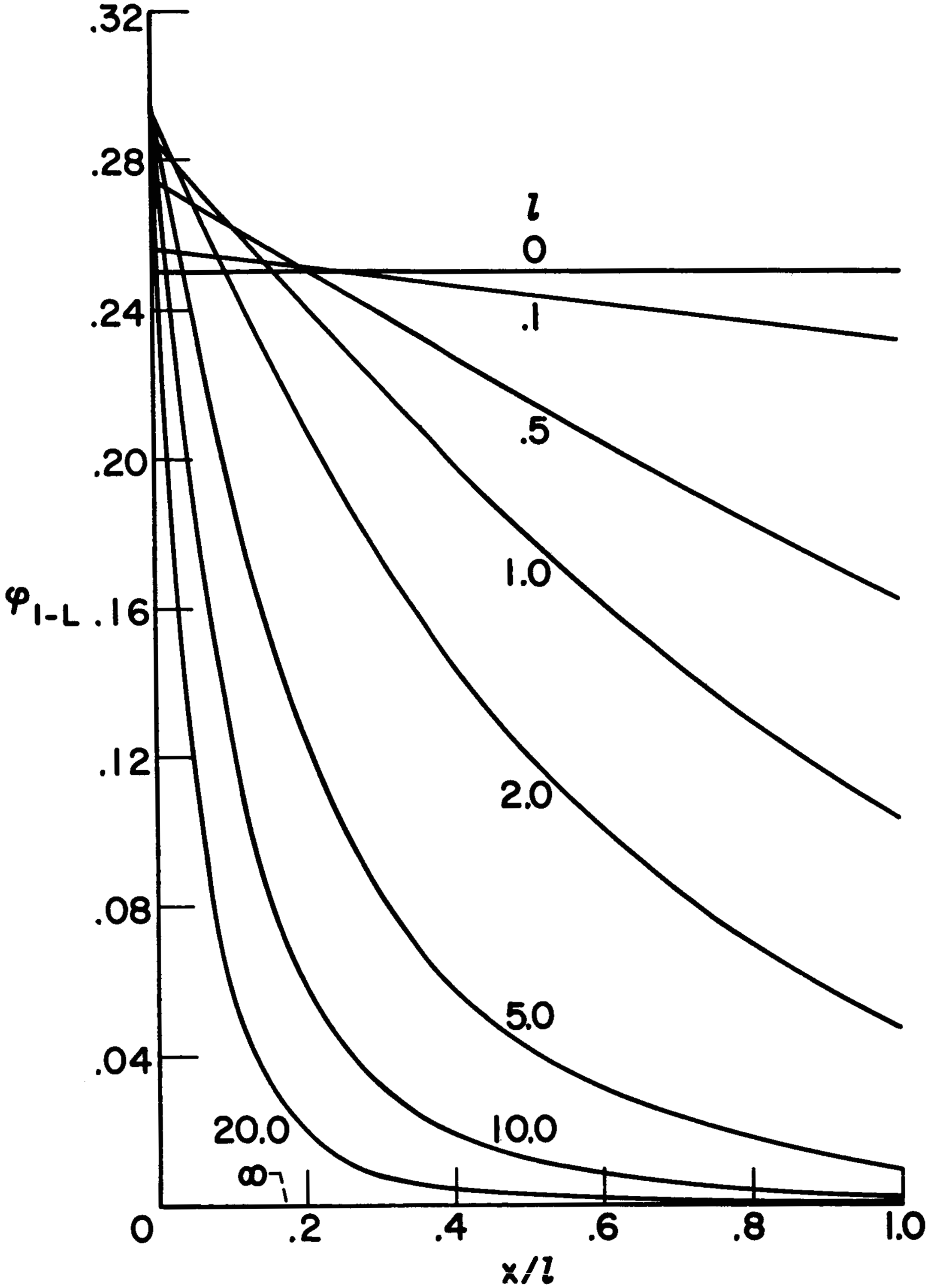
Fig. 4. - Subsolutions $\varphi_{1-1B} \equiv \left(\frac{e_{x1,t} - e_R}{m_R(E_1 - E_R)}\right)_{1-1B}$ and $\varphi_{1-2B} \equiv \left(\frac{e_{x2,t} - e_R}{m_R(E_1 - E_R)}\right)_{1-2B}$.



(b) Accommodation coefficient, $\alpha = 0.9$.

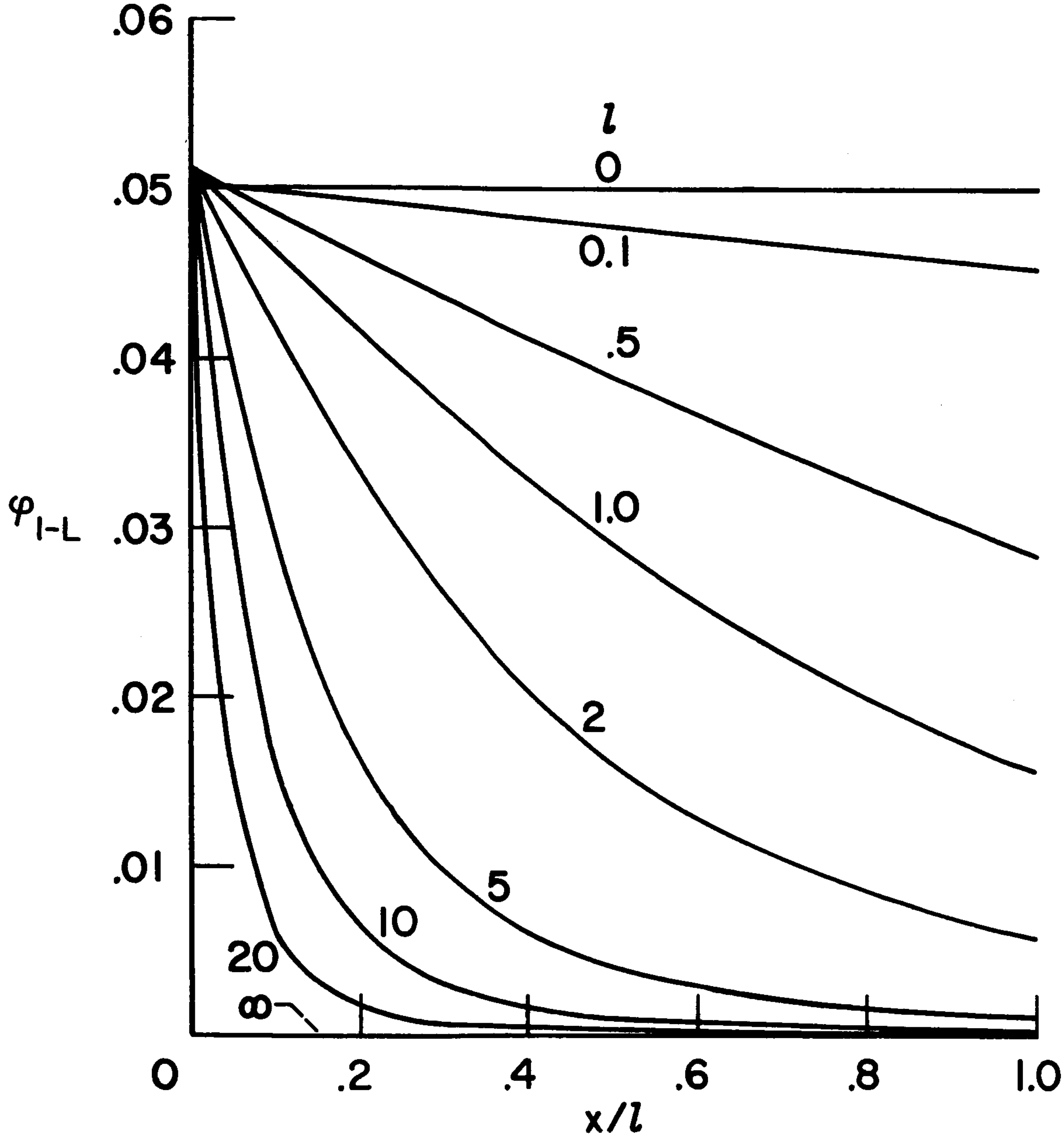
Fig. 4. - Concluded. Subsolutions $\varphi_{1\text{-}1B} \equiv \left(\frac{e_{x1,t} - e_R}{m_R(E_1 - E_R)}\right)_{1\text{-}1B}$ and $\varphi_{1\text{-}2B} \equiv \left(\frac{e_{x2,t} - e_R}{m_R(E_1 - E_R)}\right)_{1\text{-}2B}$.

E-2215



(a) Accommodation coefficient, $\alpha = 0.5$.

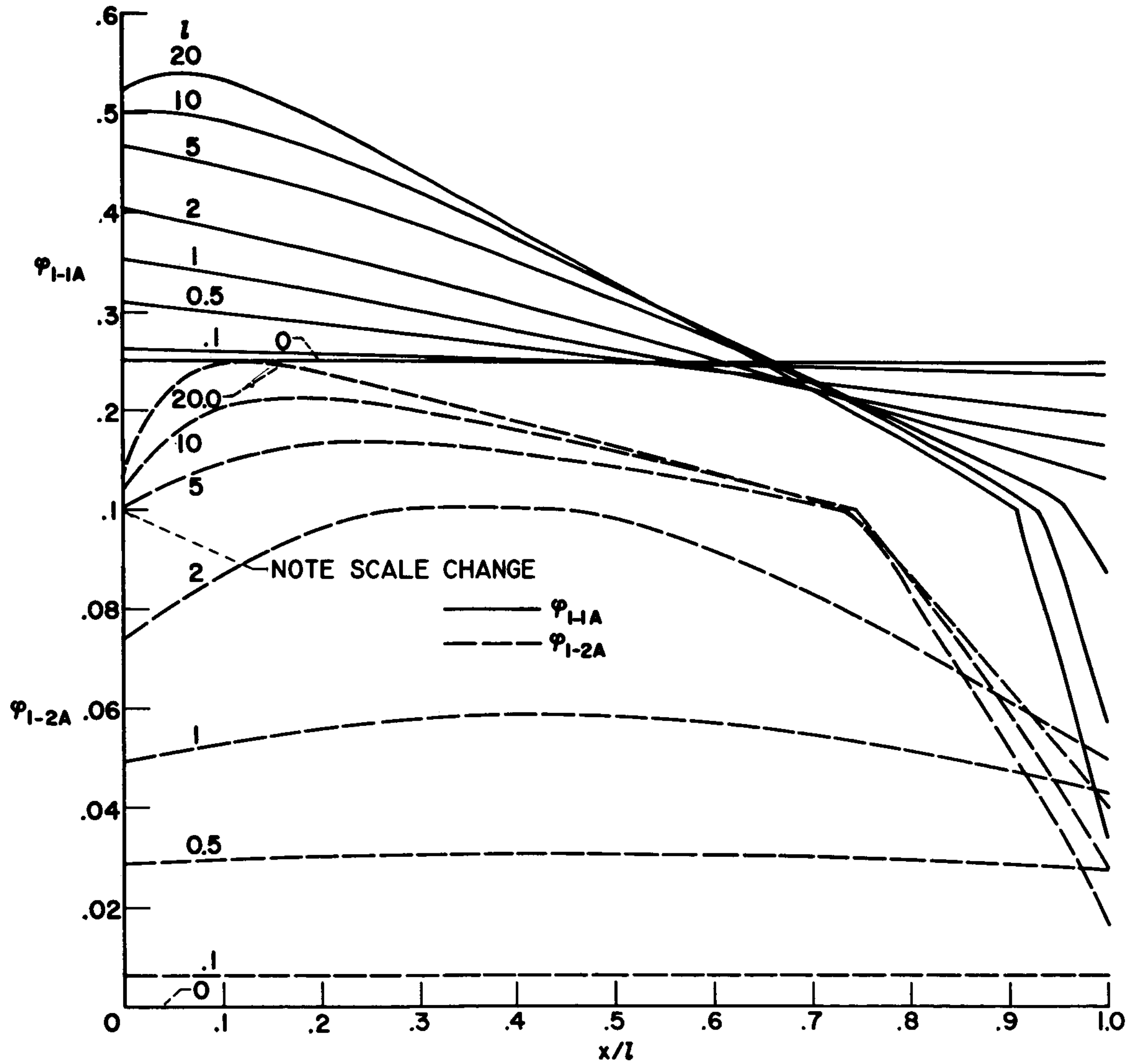
Fig. 5. - Solution $\varphi_{1-L} \equiv \left(\frac{e_{x1,t} - e_R}{e_L - e_R}\right)_{1-L}$.



(b) Accommodation coefficient, α = 0.9.

Figure 5. - Concluded. Solution $\varphi_{1-L} \equiv \left(\frac{e_{xl,t} - e_R}{e_L - e_R}\right)_{1-L}$.

E-2215



(a) Accommodation coefficient, $\alpha = 0.5$.

Fig. 6. - Subsolutions $\varphi_{1-1A} \equiv \left(\frac{e_{x1,t} - e_R}{E_1(m_L - m_R)}\right)_{1-1A}$ and $\varphi_{1-2A} \equiv \left(\frac{e_{x2,t} - e_R}{E_1(m_L - m_R)}\right)_{1-2A}$.

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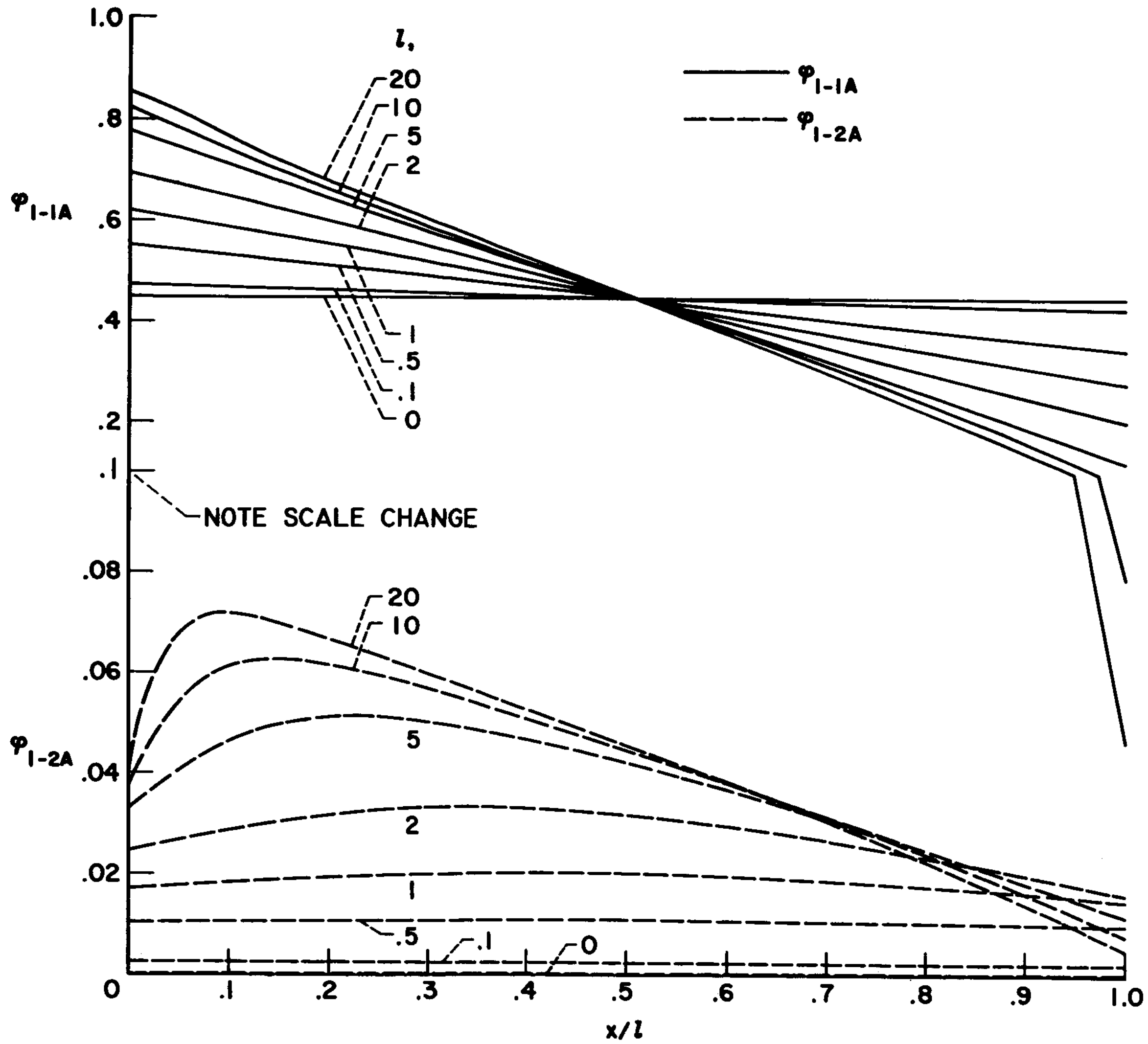
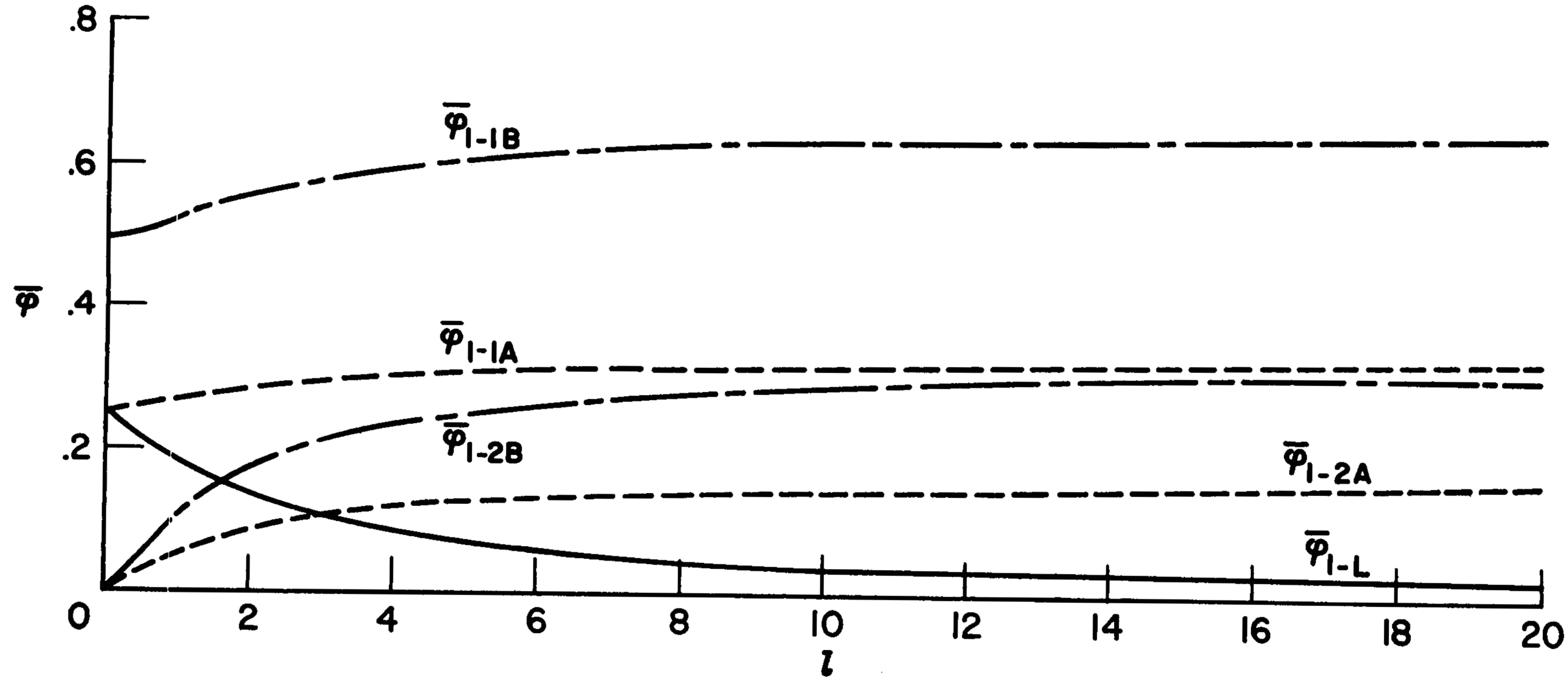


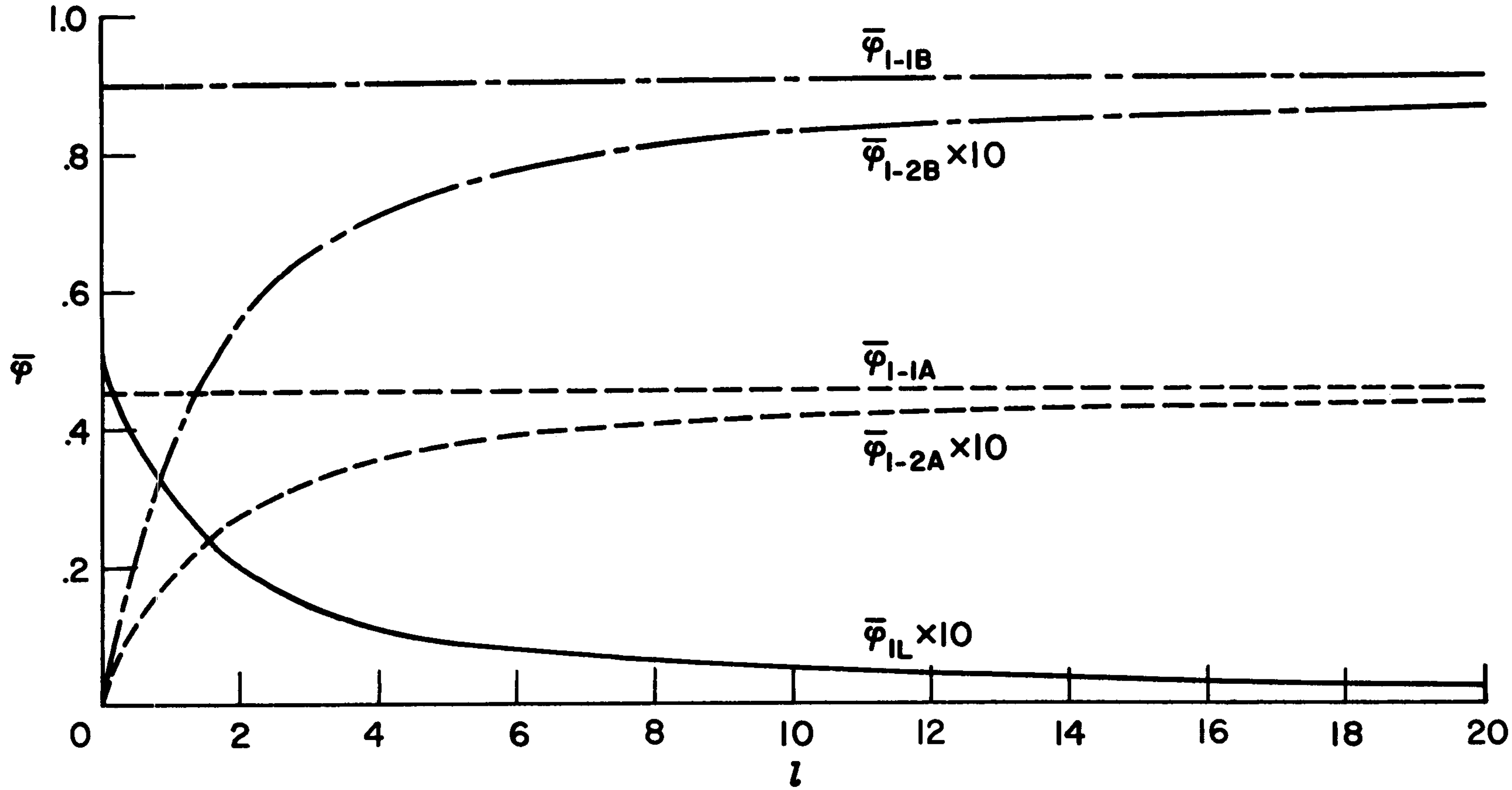
(b) Accomodation coefficient, α = 0.9.

Fig. 6. - Concluded. Subsolutions $\varphi_{1-1A} \equiv \left(\frac{e_{x1,t} - e_R}{E_1(m_L - m_R)}\right)_{1-1A}$ and $\varphi_{1-2A} \equiv \left(\frac{e_{x2,t} - e_R}{E_1(m_L - m_R)}\right)_{1-2A}$.



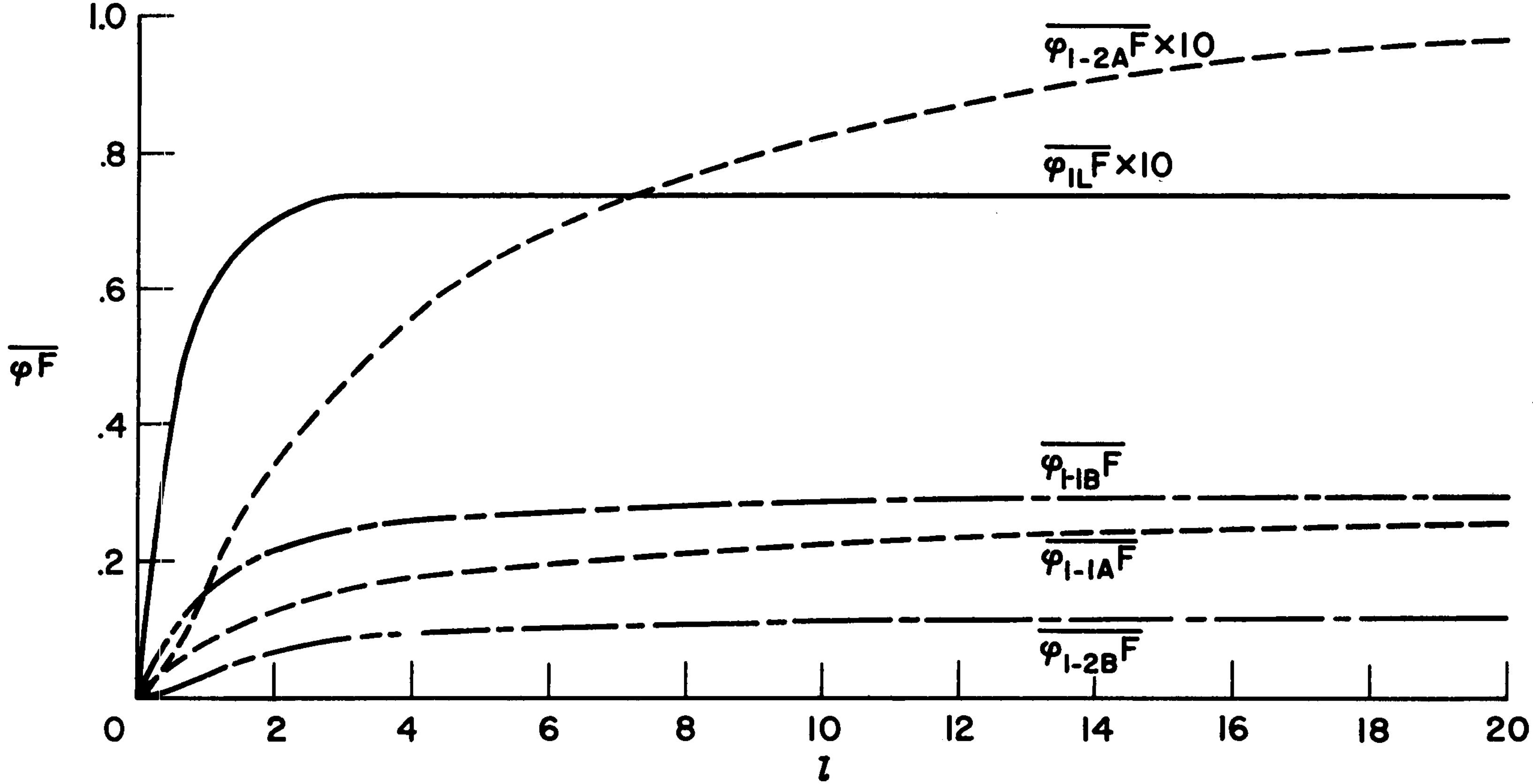
(a) Accommodation coefficient, $\alpha = 0.5$.

Fig. 7. - Integrated mean values of subsolution $\bar{\varphi} = \frac{1}{l} \int_0^l \varphi \, dx$.



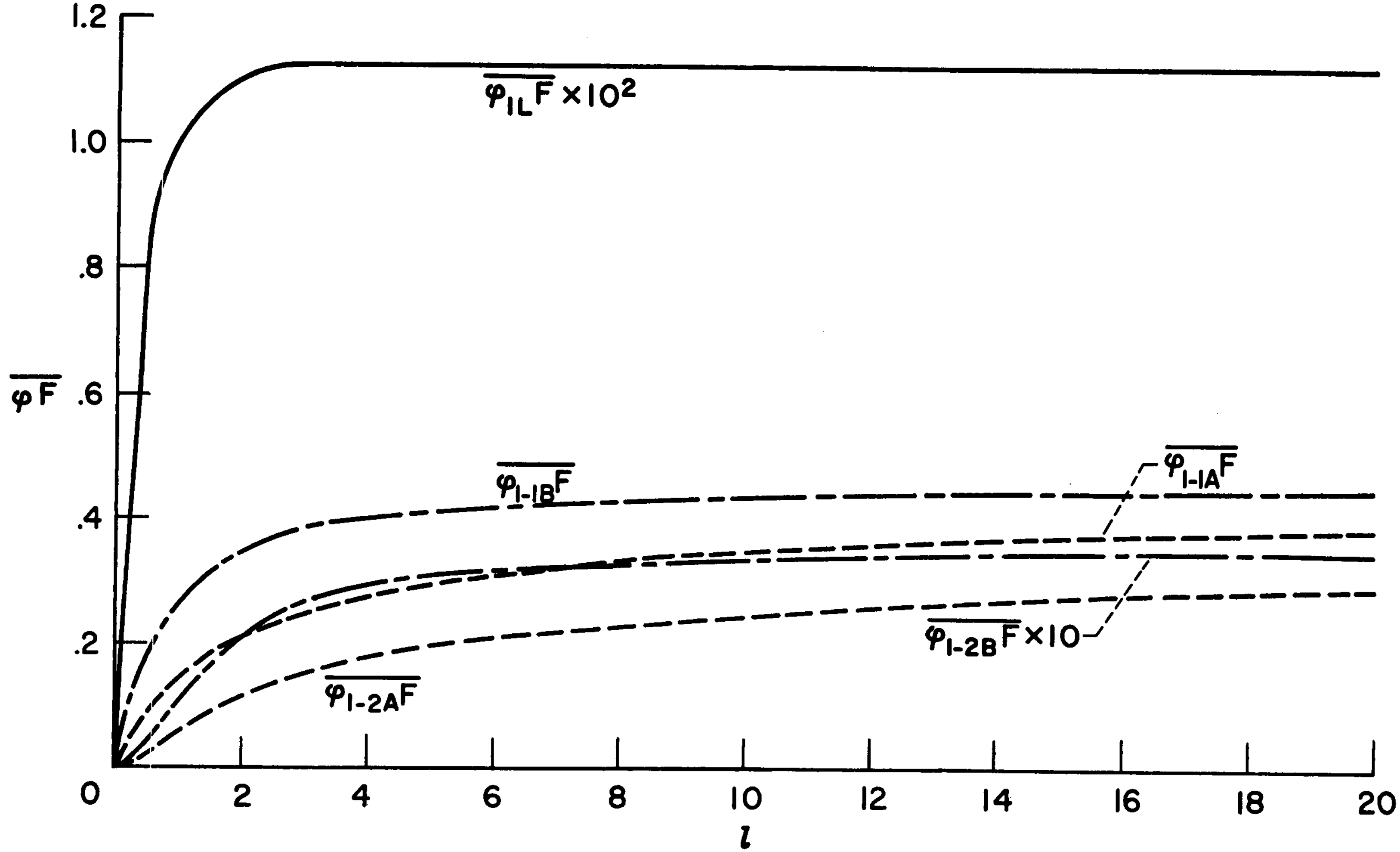
(b) Accommodation coefficient, $\alpha = 0.9$.

Fig. 7. - Concluded. Integrated mean values of subsolution $\bar{\varphi} = \frac{1}{l}\int_0^l \varphi \, dx$.



(a) Accommodation coefficient, $\alpha = 0.5$.

Fig. 8. - Integrated values $\overline{\varphi F} = \int_0^l \varphi F \, dx$.



(b) Accommodation coefficient, $\alpha = 0.9$.

Fig. 8. - Concluded. Integrated value $\overline{\varphi F} = \int_0^l \varphi F \, dx$.